

## Exercise 28

For the following exercises, use synthetic division to find the quotient.

$$(6x^3 - x^2 + 5x + 2) \div (3x + 1)$$

### Solution

Synthetic division only works if the divisor is of the form  $x - k$ . Multiply the numerator and denominator by  $1/3$  to make it so.

$$\frac{6x^3 - x^2 + 5x + 2}{3x + 1} = \frac{6x^3 - x^2 + 5x + 2}{3x + 1} \times \frac{1}{3} = \frac{2x^3 - \frac{1}{3}x^2 + \frac{5}{3}x + \frac{2}{3}}{x + \frac{1}{3}}$$

Solving  $x + \frac{1}{3} = 0$  gives  $x = -\frac{1}{3}$ ; this is the number that goes on the top left. Write out all the coefficients of the dividend to the right.

$$\begin{array}{r|rrrr} 1 & & & & \\ -\frac{1}{3} & & & & \\ \hline 3 & & & & \\ \hline & 2 & -\frac{1}{3} & \frac{5}{3} & \frac{2}{3} \end{array}$$

Bring down the leading coefficient.

$$\begin{array}{r|rrrr} 1 & & & & \\ -\frac{1}{3} & & & & \\ \hline 3 & & & & \\ \hline & 2 & -\frac{1}{3} & \frac{5}{3} & \frac{2}{3} \\ & & 2 & & \end{array}$$

Multiply the top left number by the number brought down and put the result under the second coefficient of the dividend.

$$\begin{array}{r|rrrr} 1 & & & & \\ -\frac{1}{3} & & & & \\ \hline 3 & & & & \\ \hline & 2 & -\frac{1}{3} & \frac{5}{3} & \frac{2}{3} \\ & & 2 & & \\ & & -\frac{2}{3} & & \end{array}$$

Add the numbers in the second column.

$$\begin{array}{c|ccc} \frac{1}{3} & 2 & -\frac{1}{3} & \frac{5}{3} & \frac{2}{3} \\ \hline -\frac{1}{3} & & & & \\ \hline & & \frac{2}{3} & & \\ & & -\frac{2}{3} & & \\ & & & & \\ \hline & 2 & -1 & & \end{array}$$

Multiply this sum of the second column by the top left number and put it in the next column.

$$\begin{array}{c|ccc} \frac{1}{3} & 2 & -\frac{1}{3} & \frac{5}{3} & \frac{2}{3} \\ \hline -\frac{1}{3} & & & & \\ \hline & & \frac{2}{3} & \frac{1}{3} & \\ & & -\frac{2}{3} & \frac{1}{3} & \\ & & & & \\ \hline & 2 & -1 & & \end{array}$$

Add the numbers in the third column.

$$\begin{array}{c|ccc} \frac{1}{3} & 2 & -\frac{1}{3} & \frac{5}{3} & \frac{2}{3} \\ \hline -\frac{1}{3} & & & & \\ \hline & & \frac{2}{3} & \frac{1}{3} & \\ & & -\frac{2}{3} & \frac{1}{3} & \\ & & & & \\ \hline & 2 & -1 & 2 & \end{array}$$

Multiply this sum of the third column by the top left number and put it in the next column.

$$\begin{array}{r|rrrr} \frac{1}{3} & 2 & -\frac{1}{3} & \frac{5}{3} & \frac{2}{3} \\ -\frac{1}{3} & & \frac{2}{3} & \frac{1}{3} & -\frac{2}{3} \\ \hline & 2 & -1 & 2 & \end{array}$$

Add the numbers in the fourth column.

$$\begin{array}{r|rrrr|r} \frac{1}{3} & 2 & -\frac{1}{3} & \frac{5}{3} & \frac{2}{3} \\ -\frac{1}{3} & & \frac{2}{3} & \frac{1}{3} & -\frac{2}{3} \\ \hline & 2 & -1 & 2 & 0 \end{array}$$

This final result is the remainder, and the numbers to the left are the coefficients of the quotient, which is  $2x^2 - x + 2$ .

$$(6x^3 - x^2 + 5x + 2) \div (3x + 1) = 2x^2 - x + 2$$